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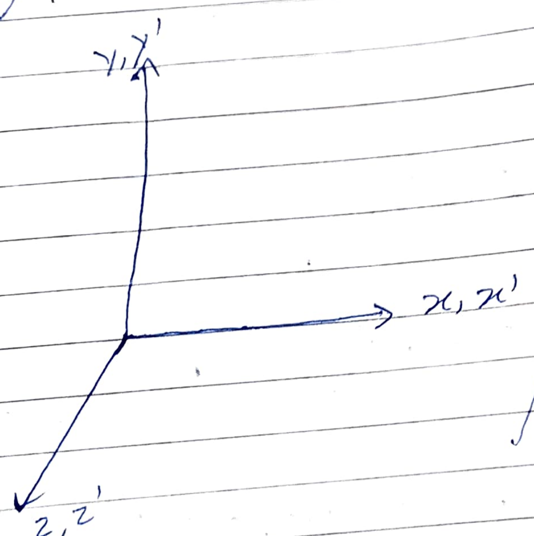
Derivation of length contraction

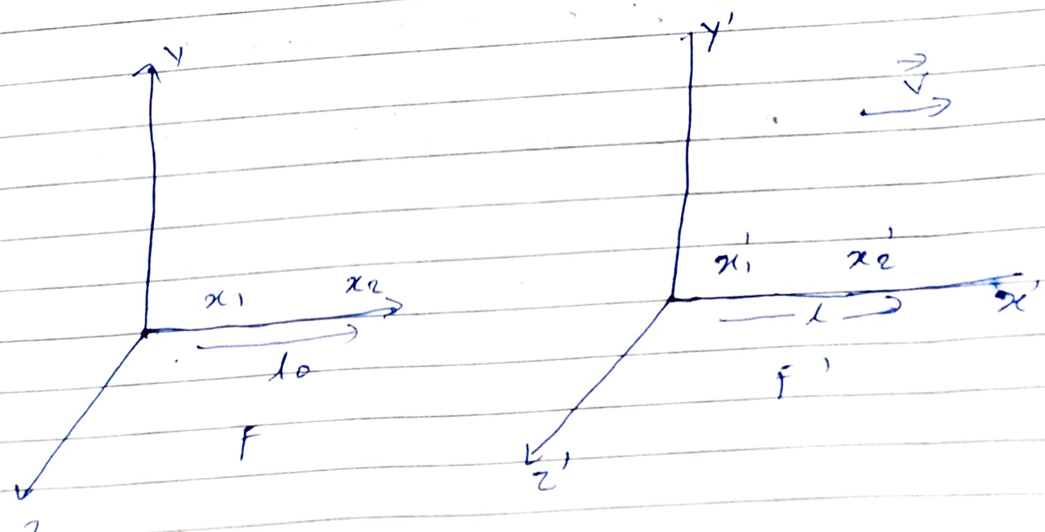
We can derive the equation for length contraction using Lorentz transformation.

Consider two reference frames F and F' that at time $t = t' = 0$ seconds are superimposed.

As time progresses, Frame F' moves to the right along x-axis with velocity $\vec{v}$.

① 

At $t = t' = 0$, the frames are superimposed.

② 

... object with length l_0 is placed ... x-axis in frame F. The co-ordinate points of the object is x_1 and x_2 in frame F,

(a) $l_0 = x_2 - x_1$

Now if that same object was found in frame F', the co-ordinate points would be x_1' and x_2'. If the length of object in frame F' is l then

(b) $l = x_2' - x_1'$

By Lorentz Equation

$$x = \alpha (x' + Vt') , \quad \alpha = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}$$

$$l_0 = x_2 - x_1 = \alpha (x_2' + Vt_2') - \alpha (x_1' + Vt_1')$$

$$= \alpha (x_2' - x_1' + V_2 t_2' - Vt_1')$$

Because $t_2' - t_1' \Rightarrow l_0 = \alpha (x_2' - x_1') = \alpha l$

$$\Rightarrow l = \frac{l_0}{\alpha} = \boxed{l_0 \sqrt{1 - \frac{V^2}{c^2}}}$$

— x —